**Lecture 31 Gratings in Integrated and Fiber Optics**

*Grating assisted coupler*

Previously we have considered only the perturbations of dielectric constant that were a function of transverse coordinates, *x* and *y.* Now we consider perturbation that is also a function of propagation coordinate *z:*  as in Fig.31.1a where it is applied to the directional couple studies in Lecture 30.



**Figure 31.1.** (a) grating assisted coupler (b) Square wave grating profile

Similar to (30.20) we can write



where subscript “g” stands for grating. Following Lecture 30 we introduce coupling coefficient as



Where



The coupled equations are



We now consider periodic index perturbation with period as shown in Fig.31.1a



which means that is also periodic



And can be expanded into Fourier series



where  and



For the “square pulse” grating shown in Fig.31.1b .

Substituting into we obtain



If Δβ is large all the terms on the RHS will rapidly oscillate and integrate to zero, unless in one of them the term in the exponent small. For the first order resonance the (Bragg) condition is



or . Leaving only this term we obtain



Where we have introduced



The solution of is known from Lecture 30 (Eq 30.72)



where



and is shown in Fig.31.2a . The beat length is



Next we express the phase mismatch as a function of frequency . Assume that perfect match is achieved at some frequency :



Expand the propagation constant in a power series



where group index is



Let the length be 1/2 beat length at ω0  :



Then



where we have introduced the bandwidth



Also



And so,



The output spectrum is shown in Fig. 31.2b



**Figure 31.2** (a) Power transfer in the grating assisted coupler (b) Frequency spectrum

The grating assisted coupler thus can perform as a filter. If we introduce transit times , the bandwidth is



Essentially it is the same as FSR of MZI of Lecture 30, inversely proportional to the length. To get a narrow filter long length is required since is small.

One application where these so-called “long period gratings” have excelled is in the fiber optics where two couple modes are the propagating mode in the core of fiber and the cladding mode as shown in Fig.31.3a. The grating itself can be designed to contain more than one period and therefore the spectrum of the incoming light can be changed. This is often used in the so-called gain-flattening filters fro erbium doped fiber amplifiers (EDFA) that makes the spectrum flatter as shown in Fig.31.3b



**Figure 31.3** (a) gain flattening long period grating filter (b) its performance

*Coupling of counter propagating waves*

Consider now the case when two waves are counter-propagating





**Figure 31.4** Bragg grating

Wavevector mismatch is  therefore the grating period should be such that



where is small. The coupled wave equations can be written as



(note the minus sign in the second equation – backward propagation). The grating coupling constant is



and it is the first Fourier components that provide coupling



Introduce new phase-shifted variables



And substitute into to obtain



The primes can be dropped and the solution sought as



Substituting into we obtain



As always, determinant needs to be zero and the characteristic equation is



With roots



Notice the change of sign under the square root compared to which has important consequence: as long as is imaginary and one can re-write as



where



Now, for some frequency, the Bragg condition is satisfied and



Then for an arbitrary frequency , following we obtain



Note that the term is much larger than  in for the long period grating.

Consider what happens at 



Change of sign compared to familiar case of co-propagating wave means that in place of trigonometric functions hyperbolic functions shall be involved and the solution is



The boundary condition is a bit tricky – we assume that the light is incident from the left and that no light is incident from the right, i.e.



Then



And the solution is



For the powers we obtain



as plotted in Fig. 31.5a, Note that



Is constant meaning the energy conservation the power lost by forward wave is transferred to the backward wave. The reflectivity of this so-called distributed Bragg reflector (DBR) is



as shown in Fig.31.5b vs the length of grating. As one can see the reflectivity is high once . Of course, Bragg grating is very similar to the multilayer interference coating consisting in quarter wave layers. Indeed each half period of Bragg grating,  is equal to the one quarter of the effective wavelength 



**Figure 31.5.** (a) Power coupling in Bragg grating (b) DBR reflectivity

*DBR Reflectivity spectrum*

Let u snow return to the case of non-zero , i.e. Eq. and use an additional substitution



So that



At which point we can drop prime. Solution is



where is defined in , and substituting the boundary conditions we obtain



so that



The reflected wave is then



and the reflectivity is



As one can see, as long as and is large the reflectivity remains high, but once the hyperbolic functions change into trigonometric and the response starts oscillating. Now use to express



Where the width of the reflection band (photonic bandgap is you want to be fancy) is



Since for high reflectivity we need  the width is roughly i.e. the inverse of the propagation time (basically an uncertainty relation). The important thing is that the width of reflection band can be engineered art will. The spectra of the DBR reflectivity for different are shown in Fig. 31.6 You can see that as increases the reflectivity becomes constant and nearly 100% over entire reflection band and then there is a strong ripple outside the high reflectivity region. The ripple can be reduced by apodizing the grating, i.,.e increasing gradually.



**Figure 31.6.** DBR reflection spectra for different strength-length products

*Types of Bragg gratings*

There are many different ways to fabricate Bragg Grating – one is using surface corrugation (Fig.31.7a), the other using sidewall corrugation (Fig.31.7b), yet another one uses periodically placed posts (Fig.31.7c) All of them give simialr results



**Figure 31.7**. Various waveguide Bragg grating (a) surface corrugated (b) wall corrugated (c) periodic posts

*Fiber Bragg gratings*

Fiber Bragg gratings (FBG) ar every widely used, FBG is essentially a notch filter as shown in Fig.31.8a. In Fig.31.8b the fabrication of FBG using the photosensitive fiber that changes refractive index where exposed to UV light is shown. FBG can be used as the ADF (add-drop filter) in communications, as shown in Fig.31.8c. Also, when fiber gets stretched the period of FBG changes and therefore the reflection band shifts which allows one to use FBFG as a strain sensor (Fig.31.8d) . It can similarly by used to sense temperature and pressure, or even a chemical composition (when the core is exposed)



**Figure 31.8 (**a) FBG fabrication (b) FBG as a notch filter (c) FBG in Add drop filter. (d FBG strain sensor

*Phase delay of Bragg grating*

The expression for the amplitude reflectivity of Bragg grating according to is



Hence the phase delay is



The frequency dependence of phase delay is shown in Fig. 31.9 . A few important facts are:

1. At the Brag frequency 
2. AT the reflection band edge 



(the last expression is for strong grating)



**Figure 31.9** Phase delay of Bragg grating for different values of 

1. For the strong grating in general



*Group delay and effective length of DBR*



**Figure 31.10 (**a) group delay in a waveguide (b) group delay in DBR (c) Equivalent waveguide with the same group delay as DBR

Consider an arbitrary waveguide with the propagation constant  and length as shown in Fig. 31.10a Then, if the input is , the output is



where and phase delay is



The *group delay* is



is the time that the light spends inside the guide. Group and phase delays is pretty much all we need to know about the waveguide to describe the optical signal propagating through it. Now consider the group delay of Bragg grating as shown in Fig. 31.10b.



Where is described by . Now we can use the equivalent reflector (“black box”) approach by introducing the length of the uncorrugated waveguide with a reflector at the end (Fig.31.10c) such that the reflected wave experiences the same group delay as in distributed Bragg reflector, i.e.



So that the effective length of DBR is



Transferring to angular frequency…



We take the derivative



And substitute it into



as shown in Fig.31.11a. Note that for low reflectivity grating , which of course make sense since for weak grating half of reflected light gets reflected before L/2 and the other half after as one can see from Fig.31.11b. But for the strong grating with , and it does not depend on actual length of the grating since very little light penetrates past 



**Figure 31.11** (a) Effective length vs. grating’s strength (b) Interpretation of Effective length

*Bragg grating as a photonic bandgap structure*

Let us now attempt to find the “supermodes” of the Bragg grating, i.e. the modes that have both forward and backward propagating components, analogous to (30.83), look for the solution of Maxwell’s equations in the form



where . The value of propagation constant is to be determined, and it is different from which is the propagation constant in the waveguide without grating. We can re-write as



According to , then



Introduce



Then we can write



where



In other words, we have forward and backward waves propagating with  whose amplitudes are given by . Substitute into and obtain



or



Once again, nontrivial solution requires zero determinant in , so the characteristic equation is



And we obtain the propagation constant



where corresponds to Bragg condition. As expected, the propagation constant for becomes complex, corresponding to high reflectivity as propagating wave cannot exist under these conditions. Now, substitute and obtain the dispersion relation



Where the optical bandgap bandwidth is . One can also write this equation as



The dispersion is plotted in Fig. 31.12a As expected there is a bandgap, region where no light can propagate centered at . Far from resonance we have - dispersion is approximately linear, i.e. grating has very little effect. But near the edge of the gap



i..e dispersion is parabolic. Note that close to the bandgap group velocity is small and at the very edge the solution is a standing wave.





**Figure 31.12** (a) Dispersion in Bragg grating (b) Standing modes on the edge of bandgap

Let us see how the modes look like at Bragg resonance . From we have



and substituting it into we obtain . Therefore, for the higher frequency solution with , i.e. we have, according to



and for the lower frequency solution we have



So we have two stranding waves. If we plot their energy densities  in Fig. 31.12 b, we can see that the lower frequency mode tends to occupy higher refractive index regions while the higher frequency mode concentrates in the lower index regions.

The fact that at frequencies within photonic bandgap the waves are exponentially decaying does not mean that they do not exist. If one has any type of defect in perfect grating, there will be a “localized” wave there. For instance, if there is a π phase shift in the grating, i.e. one of the half-periods is extended to to as in Fig, 31.13 a, there will be a mode confined inside the grating, just as in Fabry-Perot cavity and the transmission will have a maximum as shown in reflectivity spectrum of Fig.31.13 b



**Figure 31.13** (a) Localized mode in π phase shifted grating (b) Reflectivity spectrum showing a dip at the resonance.

*Chirped fiber Bragg grating (FBG)*

If the period of grating is changed as a function of length as shown in Fig. 31.14 a different wavelengtsh will have different group delays. Group velocity dispersion can be positive or negative and used to compensate the dispersion of the optical fiber that leads to broadening of the pulses as shown in Fig.31.14b. One can also use the chirped FBG as a sensor



**Figure 31.14** (a) Chirped FBG (b) It use as a dispersion compensator

Sampled grating

What if one wants to have more than one reflection band? Then one can simply superimpose different modulating functions. One way to achieve it is to use a sampled grating shown in Fig.31.15a. In the sampled grating there are alternating regions with grating (length ) and without grating (length ), where is a “superperiod”. One can introduce a square-wave sampling function as





**Figure 31.15** (a) Sampled Bragg grating and the sampling function g(z) (b) Reflection spectrum of un-sampled grating (c) Reflection spectrum of sampled grating (d) Experimental reflection spectrum

If the coupling coefficient Bragg grating is and its “wave vector” is , then one can write for the coupling coefficient of the sampled grating



The reflectivity spectrum of the un-sampled grating is centered on the frequency corresponding to the Bragg condition



i.e.  as shown in Fig. 31.15.b. Now, the sampling function can be expanded into Fourier series



Where the Fourier coefficients are



Substituting into we obtain



So, what we have is a superposition of many Bragg gratings with wave vectors



Obviously now there will be many narrow reflection bands centered at frequencies , where



as shown in Fig.31.15.c. The experimental reflectivity spectrum is shown in Fig.31.15.d

Sampled gratings are used as reflectors in tunable semiconductor lasers, using Vernier effect. This is shown in Fig. 31.16 where the active region of the laser with gain is placed between two sampled gratings with reflectivities  and . Since two sampled DBR’s have slightly different super-periods, both of them reflect simultaneously only on one wavelength  as also shown in Fig.31.16., hence laser will operate at that wavelength. Changing index in one of the reflectors on can force the shift of the reflection bands and now the laser will be forced to shift its wavelength to 



**Figure 31.16** Tunable semiconductor laser with sampled DBR’s

*DBR with gain – distributed feedback (DFB) lasers*

What one incorporates modal gain in the Bragg grating? This is shown in Fig.31.17 a where the grating is on top of the semiconductor laser active region. The two counter propagating waves now have both real and imaginary parts of propagation constants



And the mismatch is according to is



Then, according to



The reflectivity is



Now the denominator in can become quite small and when the reflector becomes amplifying, and, when denominator goes to zero, the laser reaches the oscillation threshold and starts lasing. This is shown in Fig.31.17 b, where we vary ,i.e, using we vary , and which shows that lasing occurs near the two edges of bandgap, i..e the region of the “slow wave” in Fig.31. 12. The profile of energy density inside the DFB laser is shown in Fig.31.17c



**Figure 31.17** (a) DFB laser (b) Reflectivity spectrum for different gain values. (c0 Field distribution inside DFB laser